

# Decay of low-lying $^{12}\text{C}$ resonances within a $3\alpha$ cluster model

R. Álvarez-Rodríguez<sup>1</sup>, A. S. Jensen<sup>1</sup>, D. V. Fedorov<sup>1</sup>, H. O. U. Fynbo<sup>1</sup>  
and E. Garrido<sup>2</sup>

<sup>1</sup> Department of Physics and Astronomy, University of Aarhus DK-8000 Aarhus C, Denmark

<sup>2</sup> Instituto de Estructura de la Materia, Consejo Superior de Investigaciones Científicas  
E-28006 Madrid, Spain

E-mail: raquel@phys.au.dk

**Abstract.** We compute energy distributions of three  $\alpha$ -particles emerging from the decay of  $^{12}\text{C}$  resonances by means of the hyperspherical adiabatic expansion method combined with complex scaling. The large distance continuum properties of the wave functions are crucial and must be accurately calculated. The substantial changes from small to large distances determine the decay mechanisms. We illustrate by computing the energy distributions from decays of the  $1^+$  and  $3^-$ -resonances in  $^{12}\text{C}$ . These states are dominated by direct and sequential decays into the three-body continuum respectively.

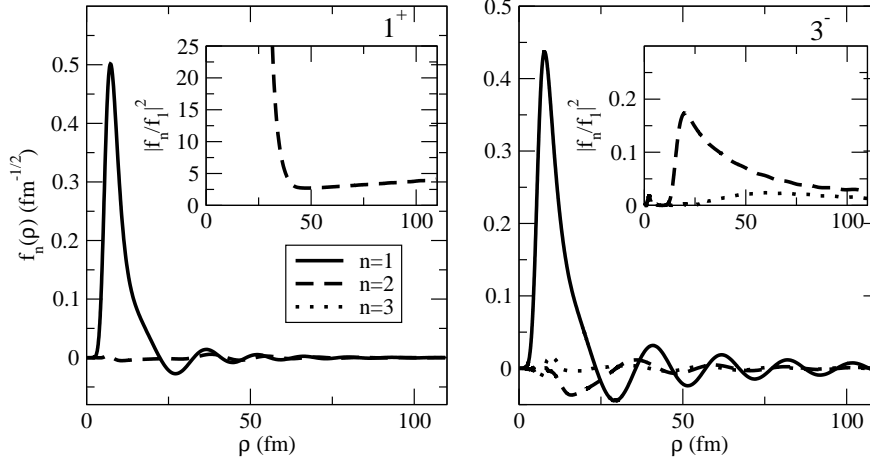
## 1. Introduction.

The low-lying resonance states of  $^{12}\text{C}$  have been studied over many years both theoretically and experimentally, motivated, in part, for its astrophysical importance [1]. Surprisingly, the energies and structure of the low-lying resonances below 16 MeV are still not well known.

We investigate in this contribution the decay of low-lying resonance states into three particle final states for the case of  $^{12}\text{C}$ , assuming that the decay mechanism is independent of how the initial state was formed. We describe the decay in analogy with  $\alpha$ -decay, assuming that the three fragments are formed before entering the barrier at sufficiently small distances to allow the three-body treatment. The hyperradius  $\rho$  provides a measure of distances for our three-body problem. Outside the range of the strong interaction, only the Coulomb and centrifugal barriers remain, since we have assumed that the small-distance many-body dynamics is unimportant for the process.

The same kind of three-body techniques could be used in other astrophysical interesting processes, e.g., the triple alpha process ( $3\alpha \rightarrow ^{12}\text{C} + \gamma$ ), the formation of  $^9\text{Be}$  ( $2\alpha + n \rightarrow ^9\text{Be} + \gamma$ ), or the 2 proton capture involved in the rp-process [2].

By measuring the properties of the particles after the decay, we get double information, as for example the energy distributions and information about the decay mechanism, e.g. direct or sequential [3]. The theoretical information about distributions of relative energies is contained in the large distance part of the resonance wave function. Numerically converged results in the appropriate region of  $\rho$ -values are then needed in order to have a reliable computation.



**Figure 1.** The adiabatic radial wave functions and the ratios of the different components (inset) for the  $1^+$  (left) and  $3^-$  (right) resonances of  $^{12}\text{C}$  at an excitation energy of 12.7 MeV and 9.6 MeV respectively.

## 2. Theoretical framework.

The decay of a  $^{12}\text{C}$  resonance into 3  $\alpha$  particles is a pure three-body problem of nuclear physics. Therefore we describe  $^{12}\text{C}$  as a  $3\alpha$ -cluster system. Moreover triple- $\alpha$  decay is the only open non-electromagnetic decay channel for this nucleus. We use the Faddeev equations and solve them in coordinate space using the adiabatic hyperspherical expansion method [4]. In order to obtain the resonances we use the complex scaling method. The so-called hyperradius  $\rho$  is the most important of the coordinates, and is defined as

$$m_N \rho^2 = \frac{m_\alpha}{3} \sum_{i < j}^3 (\mathbf{r}_i - \mathbf{r}_j)^2 = m_\alpha \sum_{i=1}^3 (\mathbf{r}_i - \mathbf{R})^2, \quad (1)$$

for three identical particles of mass  $m_\alpha = 4m_N$ , where we choose  $m_N$  to be equal to the nucleon mass,  $\mathbf{r}_i$  is the coordinate of the  $i$ -th particle and  $\mathbf{R}$  is the centre-of-mass coordinate.

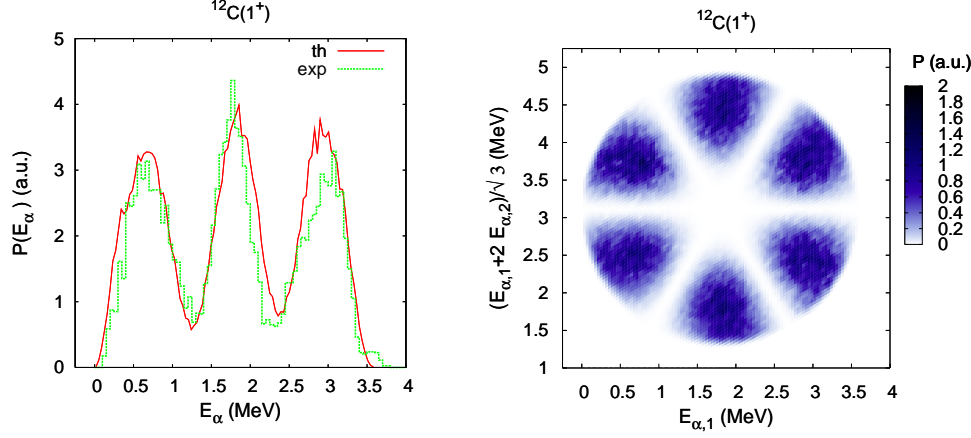
We first must determine the interaction  $V_i$  reproducing the low-energy two-body scattering properties. In this case, we have chosen an Ali-Bodmer potential [5] slightly modified in order to reproduce the s-wave resonance of  $^8\text{Be}$ . We add then a three-body potential, whose range corresponds to three touching  $\alpha$ -particles. These potentials are chosen independently for each  $J^\pi$ .

The total wave function is expanded on the angular eigenfunctions  $\Phi_{nJM}(\rho, \Omega)$  obtained as solutions to the Faddeev equations for fixed  $\rho$ ,

$$\Psi^{JM} = \frac{1}{\rho^{5/2}} \sum_n f_n(\rho) \Phi_{nJM}(\rho, \Omega), \quad (2)$$

where the radial coefficients  $f_n(\rho)$  are obtained from the coupled set of radial equations. The effective adiabatic potentials are the eigenvalues of the angular part.

At intermediate distances the potential has a barrier that determines the resonance width; and at large distances the resonance wave functions contain information about distributions of relative energies.



**Figure 2.** The  $\alpha$ -particle energy distribution (left) and Dalitz plot (right) for the  $1^+$  resonance of  $^{12}\text{C}$  at 5.43 MeV above the  $3\alpha$  threshold at an excitation energy of 12.7 MeV. Experimental data are from [8].

The asymptotic behaviour of the decaying resonance wave function determines the energy distribution in the observable final state. This energy distribution can be computed in coordinate space, except for a phase-space factor, as the integral of the absolute square of the total wave function for a large value of the hyperradius [6]. We shall explore the conjecture that the final state can be obtained entirely within the present cluster model.

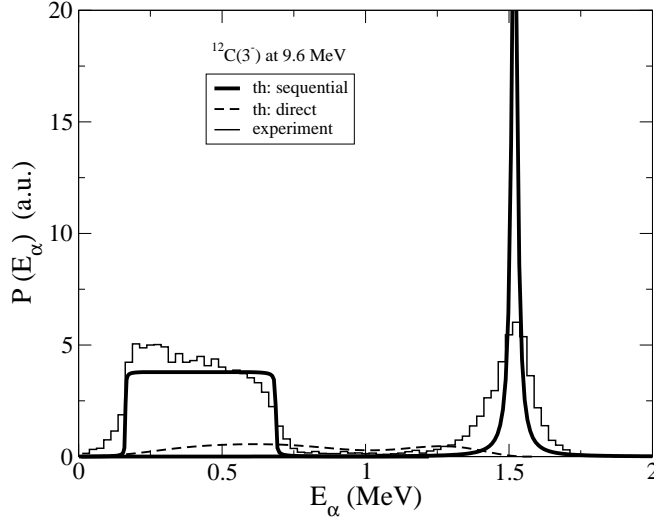
### 3. Results.

By using this method we could investigate a number of the low-lying  $^{12}\text{C}$  resonances below 15 MeV, i.e., two  $0^+$ , three  $2^+$ , two  $4^+$ , and one of each of  $1^\pm$ ,  $2^-$ ,  $3^\pm$ ,  $4^-$  and  $6^+$  [7].

In fig. 1 we have plotted the radial wave functions together with the absolute squared values of the ratios between the different components for both  $1^+$  and  $3^-$  resonances of  $^{12}\text{C}$ . One can observe that these ratios are fairly insensitive to variations of the hyperradius at large distances, that is where the energy distributions are computed. This means that for those distances the asymptotical behaviour has been already reached.

Fig.2 shows the individual  $\alpha$ -particle energy distribution and the Dalitz plot for the  $1^+$  state of  $^{12}\text{C}$  at 12.7 MeV of excitation energy (or at 5.43 MeV above the  $3\alpha$  threshold). This  $1^+$  state of  $^{12}\text{C}$  is often referred to as a shell-model state, which means that it can not be described as a cluster structure. Should the final state consists of  $3\alpha$  particles, a three-body description is unavoidable. Moreover, due to parity considerations the sequential decay via the  $^8\text{Be}$  ground state is forbidden. We have compared the theoretical energy distribution with the accurate experimental data for this resonance populated in  $\beta$ -decay [8]. Fig. 2 shows an impressive agreement between theory and experiment. If we compare the Dalitz plot with fig. 1 of the reference [9], it is clear that we are able to reproduce also the basic features of the Dalitz plot obtained from the experimental data.

The energy distribution for the natural parity state  $3^-$  at 9.6 MeV of excitation energy (or at 2.33 MeV above the  $3\alpha$  threshold) shows the characteristic features of a sequential decay via the  $^8\text{Be}$  ground state, i.e. a narrow high-energy peak, and a distribution around  $E_{max}/4$  (see fig. 3). Since the complex rotated two-body asymptotic behaviour correspond to a bound state, the computed distributions are not accurate. Instead we can exploit the fact that precisely one of the adiabatic potentials asymptotically must describe the two-body resonance and the third



**Figure 3.** The  $\alpha$ -particle energy distribution for the  $3^-$  resonance of  $^{12}\text{C}$  at 2.33 MeV above the  $3\alpha$  threshold at excitation energy of 9.6 MeV. Contributions from direct and sequential decays via  $^8\text{Be}(\text{g.s.})$  are shown. Experimental data are from [10].

particle far away. The corresponding sequential decay is then two consecutive two-body decays with the related well-known distinct kinematics. This component can then be substituted by the Fourier transform of the known asymptotic two-body behaviour, as it has been previously done in [3]. By looking at the ratios between the different adiabatic radial wave functions (fig. 1), we can estimate that the direct decay is about 4% and sequential decay via  $^8\text{Be}(0^+)$  is 96% of the total distribution. This sequential part of the energy distribution is approximated by the leading order Breit-Wigner shape for the first emitted  $\alpha$ -particle. It has a high-energy peak at the most probable position (2/3 of the resonance energy). The width is the sum of the three-body decaying resonance width and the width of the intermediate two-body resonance. By kinematic conditions we can compute the energy of the two  $\alpha$ -particles emerging from  $^8\text{Be}$ , that gives rise to the peak at lower energy. After removing the contribution from the first adiabatic potential, the remaining energy distribution is described as direct decay by the computed three-body continuum coordinate space wave function. The distribution is rather uniform but mostly visible between the separate peaks of the sequential decay. We have compared our result with some preliminary data from the reaction  $^{10}\text{B}(^3\text{He}, p\alpha\alpha\alpha)$  studied at CMAM (Madrid) by O. Kirsebom and collaborators [10]. Even though the analysis of the data is not yet finished, we find a nice agreement between the basic features of the measured and the theoretical energy distributions.

#### 4. Summary and Conclusions.

We have applied a general method to compute the particle-energy distributions of some of the three-body decaying  $^{12}\text{C}$  many-body resonances. We conjectured that the energy distributions of the decay fragments are insensitive to the initial many-body structure. The energy distributions are then determined by the energy and three-body resonance structure as obtained in a three- $\alpha$  cluster model. These momentum distributions are determined by the coordinate space wave functions at large distances. We can separate components with two- and three-body asymptotics that correspond to sequential and direct decays respectively.

We have shown the examples of  $1^+$  and  $3^-$  states of  $^{12}\text{C}$ . The  $1^+$  resonance is best described by

direct decay into the three-body continuum, whereas the  $3^-$  resonance have substantial cluster components and decays preferentially via the  $^8\text{Be}$  ground state. In both cases we have compared our computation with the measured distributions and found a nice agreement between theory and experiment.

In conclusion, we predict energy distributions of particles emitted in three-body decays. Both sequential and direct, and both short and long range interactions are treated.

### Acknowledgments

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